EECS 861
Homework 10

1. Chapter 4: Problem 4.5

2. Chapter 4: Problem 4.12

3. The transfer function of a linear time invariant filter is $H(f) = \dfrac{100}{0.001 + j2\pi f}$

   The input to this filter is zero-mean white Gaussian Noise with $S_X(f) = 10^{-10}$ Watts/Hz

   a) Find the PSD of the filter output Y(t).
   b) Find E[Y(t)]
   c) Find Var[Y(t)]
   d) Find the noise power at the filter output Y(t).

4. N(t) is bandpass white Gaussian noise with $\eta/2=10^{-14}$, centered at 10 Mhz with a bandwidth B_N = 10 Mhz.
   a. Find the noise power.
   b. Find $P(|N(t)|>5\text{x}10^{-4})$.

5. A random signal X(t) is corrupted by statistically independent additive white Gaussian noise, N(t) with a bandwidth B_N and is input to a linear time-invariant filter H(f).

   $$S_N(f) = \begin{cases} \dfrac{\eta}{2} & |f| < B_N \\ 0 & \end{cases}$$

   $X(t) = A\cos(2\pi f_c t + \theta)$ where θ is uniformly distributed $[0, 2\pi]$ and A and f_c are constants

   $$H(f) = \frac{1}{1 + j(2\pi f / f_0)}$$

   a) Find the input signal-to-noise ratio for B_N =100 kHz, $A=10^{-5}$ V, f_c=50 Hz, f_o=1000Hz, and $\eta=10^{-15}$
   b) Find the output signal-to-noise ratio for B_N =100 kHz, $A=10^{-5}$ V, f_c=500 Hz, f_o=1000Hz, and $\eta=10^{-15}$
   c) Find the output signal-to-noise ratio for B_N =100 kHz, $A=10^{-5}$ V, f_c=1000 Hz, f_o=1000Hz, and $\eta=10^{-15}$
   d) Find the output signal-to-noise ratio for B_N =100 kHz, $A=10^{-5}$ V, f_c=2000 Hz, f_o=1000Hz, and $\eta=10^{-15}$
   e) Why does the output signal-to-noise ratio change as f_c changes.

6. A first order autoregressive process is defined as $Y[k]=a_1Y[k-1] + N[k]$. Where N[k] is zero-mean white Gaussian Noise with a variance of $\sigma^2=1$. Let $a_1=-0.5$ and $\sigma^2=1$. Find $E[Y(k)]$, $E[Y^2(k)]$, $Var[Y(k)]$, and $R_{YY}(m)$.

7. Chapter 5:
    a. Problem 5.58
    b. If in Problem 5.58 f_0 is changed to f_0=995 kHz, does $S_{NcNc}(f)$, and $S_{NcNs}(f)$ change, if so why?

8. A second order moving average process is defined by $X[n] = e[n] + b_1 e[n-1] + b_2 e[n-2]$ where e[n] is zero-mean white Gaussian Noise with a variance of σ^2. Find the covariance matrix for X[n], X[n-1], and X[n-2]. For b_1 = .75 & b_2=.5 and σ^2=2 validate your result using http://www.ittc.ku.edu/~frost/EECS_861/Mathematica_files/MA-order-2-Take-1_example.cdf

9. Use the data in given file.
    a. Find and plot the autocorrelation function of this data. http://www.ittc.ku.edu/~frost/EECS_861/Mathematica_files/Homework_10_data_2017.csv
    b. Could this data be a sample function from an MA(3) process, justify your answer.
    c. Assuming this data is a sample function from an AR(1) process suggest a value for α_1. Hint experiment with the AR(1) example in http://www.ittc.ku.edu/~frost/EECS_861/Mathematica_files/ARMA_study-V4.cdf